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Normality under uncertainty

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Abstract

Consider the demand for a good whose consumption must be chosen prior to the resolution of uncertainty regarding income. How do changes in the distribution of income affect the demand for this good? In this paper we show that normality, is sufficient to guarantee that consumption increases if the Radon-Nikodym derivative of the new distribution with respect to the old is non-decreasing in the whole domain. However, if only first order stochastic dominance is assumed more structure must be imposed on preferences to guarantee the validity of the result. Finally a converse of the first result also obtains. If the change in measure is characterized by a non-decreasing Radon-Nicodym derivative, consumption of such a good will always increase if and only if the good is normal.
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1 Introduction

The problem of comparative statics under uncertainty has now a long tradition in economics, dating back at least to Dalal [6]. Most of the research agenda is concerned with answering the question of how asset allocation is affected by changes in some random variables[1]. This paper is related to this research agenda but focuses, instead, on the issue of normality under uncertainty.

To be precise, think of an agent whose preferences are defined in terms of two goods only. The amount he consumes of one of the goods must be chosen before the realization of her income. The first question we try to answer here is the following. Suppose that one replaces the original by another that dominates it in a first order stochastic dominance sense, is normality sufficient to guarantee

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[1] Eg. Dalal [6], Cheng et Alli. [3], Ormiston [7].

that the agent increases the consumption of this latter good in response to this change?

We show that this needs not be the case. Increase in the consumption of the pre-uncertainty good is only guaranteed if we either impose stronger restrictions on preferences, or restrict the change in measure to be monotonic. This latter property on the change in measure is both necessary and sufficient for any normal good to remain normal.

A converse of this result also holds. Namely, if the change in the distribution has a monotonic Radom-Nicodym derivative, than normality is necessary and sufficient for the consumption of the good to increase.

While the first set of results is important when evaluating choices that demand pre-commitment - as is the case of choosing as the number of kids, that usually comes prior to an agent realizing her own earnings potential. The second, has important applications in mechanism design, where one uses changes in demand as a signal of deviations from prescribed behavior.

2 The Model

Consider an agent whose preferences are defined as a function of two goods z and x. The agent's utility function is $u(z, x)$. Absent any uncertainty good z is normal, i.e., $dz/dy > 0$. But it is assumed that income is random and the consumption of z must be decided prior to the resolution of uncertainty[2].

We consider two distribution functions f_1 and f_2 absolutely continuous with respect to each other, i.e., two equivalent probability measures. We first show a case where the new distribution f_2 first order stochastically dominates the other but the consumption of z is lower under f_2 than under f_1.

Assume that $u(z, x)$ is a regular strictly quasi-concave function. In this case, the first order conditions characterize the maximum.

Example 1 *Take the following example. There are* 3 *possible states of the world,* ω_1, ω_2 *and* ω_3 *with probabilities* $p(\omega_i) = 1/3, i = 1, 2, 3$. *And,*

$$\begin{aligned}
\frac{u_z}{u_y}(\omega_1) &= 1/2 \text{ and } u_y(\omega_1) = 1 \\
\frac{u_z}{u_y}(\omega_2) &= 5/3 \text{ and } u_y(\omega_2) = 1/2 \\
\frac{u_z}{u_y}(\omega_3) &= 2 \text{ and } u_y(\omega_3) = 1/6
\end{aligned}$$

In this case,

$$\begin{aligned}
u_z(\omega_1) - u_y(\omega_2) &= -1/2 \\
u_z(\omega_2) - u_y(\omega_2) &= 1/3 \\
u_z(\omega_3) - u_y(\omega_3) &= 1/6
\end{aligned}$$

[2] This is how a pre-committed good is defined in Cremer and Gahvari [4]. It is also how one would think of effort (with a change of sign) in a model of moral hazard.

Then

$$E_p[u_z - u_y] = 0$$

Now suppose we use a change in measure such that $\pi(\omega_1) = 1/3$, $\pi(\omega_2) = 1/6$, $\pi(\omega_3) = 1/2$. Notice that this new distribution first order stochastically dominates the other[3]. *But $E_\pi[u_z - u_y] < 0$. Since the problem is concave, the optimum under the new distribution must have a lower level of consumption of z.*

So what we may see from this result is that first order stochastic dominance does not suffice for the consumption of pre-uncertainty good z to increase. We next restrict preference to get monotonicity.

First define

$$z^1 \equiv \arg\max_z \int_{\underline{y}}^{\overline{y}} u(z, y-z) f_1(y)\, dy \tag{1}$$

$$z^2 \equiv \arg\max_z \int_{\underline{y}}^{\overline{y}} u(z, y-z) f_2(y)\, dy \tag{2}$$

We now show that if u is supermodular[4] first order stochastic dominance is sufficient to guarantee monotonicity.

Proposition 2 *If $u_{zx} \geq 0$ then $z^2 > z^1$.*

In what follows we shall prove that if the transformation is such that the two distributions are related by a monotonic likelihood ratio property, then normality is sufficient to guarantee the increase in the consumption of z.

For this we need the following lemma.

Lemma 3 *Assume $f_2(y)/f_1(y)$ is increasing in y, where f_1 and f_2 are both differentiable and $f_2 > 0 \quad \forall y$. Let $\alpha(y)$ be a Radon-Nikodym derivative of F_2 with respect to F_1 then, if α is differentiable $\alpha'(y) \geq 0\ \forall y$.*

Since we want to determine how z^2 compares to z^1, we evaluate, at z^1 the sign of the derivative of the agents' expected utility when the relevant distribution is f_2. If this sign is positive (negative) the concavity of the problem with respect to z implies that the z^2 must be greater (smaller) than z^1.

Proposition 4 *Let z be a normal good in the absence of uncertainty. Let also z^1 and z^2 be defined by (1) and (2), then if $f_2(y)/f_1(y)$ is increasing in y:*

$$\frac{d}{dz}\left(\int_{\underline{y}}^{\bar{y}} u(z, y-z) f_2(y)\, dy\right)\Bigg|_{z^1} > 0.$$

[3] See Rothschild and Stiglitz [8] for a definition.

[4] Because we have assumed the utility to be differentiable, u is supermodular if and only if $u_{xz} \geq 0$ - see Topkis [9].

So, what comes up form this is that, first order stochastic dominance is not enough. But if the changes in measure is done in such a way as to (weakly) increase the probability of higher states more than weak states, i.e., the transformation has the monotone likelihood ratio property, than the result goes through.

This is the type of change in measure (with the monotone likelihood ratio property) we typically observe not only in moral hazard problems but, with a careful reinterpretation of deviating behavior 'ex-post', also in self-selection problems[5]. Hence, these findings provide a rationale for the subsidization results found in Cremer and Gahvari [4] and da Costa and Werning [5].

3 Related Literature

We now introduce a somewhat different notation from the one we were using up to this point. This is done to facilitate our relating the results herein to the ones found in the monotone comparative statics literature.

Consider the case where the density function is parametrized by $f(y;\theta)$, where θ is a given unidimensional parameter. The first thing to notice is that $\alpha(y)$, as defined in lemma 3, is increasing in y if and only if f is log-supermodular[6]. So, we shall no longer speak of $\alpha(\cdot)$, referring, instead, to increases in θ. This new notation will prove itself very handy, for Athey [1] has proved many important results for log-supermodular f.

We start with one her main results. Log-supermodularity of u and f is sufficient to guarantee that $U(z,\theta) \equiv \int u(z,y) f(y,\theta)\, dy$ is log also log-supermodular. Log-supermodularity of $U(z,\theta)$ would then guarantee monotonicity of z with respect to θ.

As we have already argued, the transformation we propose in proposition 4 is equivalent to increasing θ with f a log-supermodular function, however we do not have log-supermodularity of u in our problem - this property is not equivalent to normality of u. Hence, the result is not directly applicable here. Also note that whenever we relaxed the requirement on the transformation of f we added to u supermodularity (which is weaker than supermodularity if we are dealing with goods rather than bads). Therefore, the result is not applicable here.

However, we can use one of her results - lemma 8 (p.210), to be specific - in our framework, and provide an alternative proof for Proposition 4.

One important comment is that this alternative proof also allows us to see that because the condition is necessary and sufficient, one should expect that for other types of transformations there ought to exist utility functions for which that would violate monotonicity of z with respect to θ.

Though not directly related to the question we aimed at answering the next proposition is an important by-product of the applications of results on monotone comparative statics to this framework.

[5] See da Costa and Werning [5].

[6] A function f is said log-supermodular if $log(f)$ is supermodular.

Proposition 5 *Let $f(y, \theta)$ be log-supermodular. Then the set of utility functions $u(\cdot)$ for which z is increasing in θ is the one where utility functions represent preferences such that z is normal.*

To understand the meaning of such result, imagine a moral hazard problem where preferences are defined over effort and two goods, one of them being pre-committed in the sense we have already discussed.

Then, with separability between effort and these goods, if the distribution of payoff for the agent is changed in a transformation with the MLRP then agents that put lower effort decrease the consumption of good z if and only if it is normal. The implication for mechanism design is that the consumption of this good is to be 'subsidized' in the case of normality and 'taxed' in the case of inferiority.

4 Conclusion

One good is said to be normal whenever its consumption is increased with the level of income. But what if income is stochastic and the consumption of the good is to remain fixed? How would one define an 'increase in income' that guarantees that a normal good *is* normal?

This paper shows that normality is a property preserved under uncertainty, provided that the change in distribution is monotonic. If the density, f is parametrized by a scalar θ this condition amounts to the change having the monotone likelihood ratio property - or, equivalently, f being log-supermodular.

We also provide an example where first order stochastic dominance is shown not to be sufficient for guaranteeing that the demand of a normal good will increase with this type of change in the distribution.

Finally, normality is shown to be necessary and sufficient for a transformation with the monotone likelihood property to generate an increase in the consumption of f. This latter result has important implications for the design of optimal contracts under moral hazard.

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A Appendix

Proof of Proposition 2. Under this restrictions on preferences, $u_z - u_x$ is increasing in x. Thus, first order stochastic dominance implies

$$\int_{\underline{y}}^{\overline{y}} u\left(z^1, y - z^1\right) f_2\left(y\right) dy > 0.$$

Thus $z^2 > z^1$. ■

Proof of lemma 3. Since α is a Radon-Nikodym derivative we have that for every measurable function ϕ, and every measurable set E.

$$\int_E \phi\left(y\right) f_2\left(y\right) dy = \int_E \phi\left(y\right) \alpha\left(y\right) f_1\left(y\right) dy$$

In particular,

$$\int_E \frac{f_2\left(y\right)}{f_1\left(y\right)} dy = \int_E \alpha\left(y\right) dy$$

Since this is valid for every E, then $f_2/f_1 = \alpha$ almost everywhere, by the Radon-Nikodym theorem (See Bartle [2], p. 85). Notice also that under the differentiability assumption we used, the qualifier a.e. is not needed. ■

Proof of Proposition 4. The first order condition for the agent's problem if the relevant distribution is f_1 is:

$$\int_{\underline{y}}^{\overline{y}} \left(u_z\left(z^1, y - z^1\right) - u_x\left(z^1, y - z^1\right)\right) f_1\left(y\right) dy = 0 \tag{3}$$

Now, consider the term:

$$\left(\frac{u_z\left(z^1, y-z^1\right)}{u_x\left(z^1, y-z^1\right)}-1\right) u_x\left(z^1, y-z^1\right) \tag{4}$$

Since z^1 is fixed, normality of z^1 implies that $u_x\left(z^1, y-z^1\right)/u_z\left(z^1, y-z^1\right)$ -is increasing in y.
Moreover, from (3) it is clear that there is a value y^* such that $u_x/u_z > 1$ (resp. < 1) for all $y > y^*$ (resp. $y < y^*$), and $u_x\left(z^1, y^*-z^1\right)/u_z\left(z^1, y^*-z^1\right) = 1$. Consider now the derivative

$$\begin{aligned} &\frac{d}{dz}\left(\int_{\underline{y}}^{\bar{y}} u\left(z, y-z\right) f_2\left(y\right) dy\right)\Bigg|_{z^1} \\ &\qquad = \int_{\underline{y}}^{\bar{y}} \left(u_z\left(z^1, y-z^1\right) - u_x\left(z^1, y-z^1\right)\right) \alpha\left(y\right) f_1\left(y\right) dy \end{aligned}$$

The right hand side of the latter expression can be written as

$$\int_{\underline{y}}^{y^*} \left(u_z - u_x\right) \alpha f_1 dy + \int_{y^*}^{\bar{y}} \left(u_z - u_x\right) \alpha f_1 dy,$$

where the first term is negative and the second positive. Now, from lemma 3 and because the integrand does not change signs in the subintervals, we have:

$$\begin{aligned} \int_{\underline{y}}^{y^*} \left(u_z - u_x\right) \alpha f_1 dy &\geq \alpha\left(y^*\right) \int_{\underline{y}}^{y^*} \left(u_z - u_x\right) f_1 dy \\ \int_{y^*}^{\bar{y}} \left(u_z - u_x\right) \alpha f_1 dy &\geq \alpha\left(y^*\right) \int_{y^*}^{\bar{y}} \left(u_z - u_x\right) f_1 dy \end{aligned}$$

Adding the two inequalities, we get:

$$\int_{\underline{y}}^{\bar{y}} \left(u_z - u_x\right) \alpha f_1 dy \geq \alpha\left(y^*\right) \int_{\underline{y}}^{\bar{y}} \left(u_z - u_x\right) f_1 dy = 0$$

Since our problem is concave, this implies that optimum level of expenditure is greater when we use f_2. ■

To provide an alternative proof for proposition 4 and prove proposition 5 we need, first, the following result.

Lemma 6 *Define a function $h : \mathbb{R}^3 \to \mathbb{R}$ by*

$$h\left(z, x, t\right) \equiv u\left(z, x+t\right).$$

Then, z is normal if and only if h satisfies Spence-Mirrlees (SM). That is,

$$\frac{d}{dt}\left(\frac{\partial h\left(z, x, t\right)/\partial z}{\partial h\left(z, x, t\right)/\partial x}\right) \geq 0.$$

Proof. Just notice that

$$\frac{d}{dt}\left(\frac{\partial h\left(z,x,t\right)/\partial z}{\partial h\left(z,x,t\right)/\partial x}\right)=-\frac{d}{dt}\left(-\frac{u_{z}\left(z,x+t\right)}{u_{x}\left(z,x+t\right)}\right).$$

This expression being non decreasing is a necessary and sufficient condition for z to be normal. ■

Alternative Proof of Proposition 4. We are looking for transformations of f that guarantee that z increases for all $u\left(z,x\right)$ such that z is normal.

The strategy here is to define a function $V\left(z,x,\theta\right)\equiv\int h\left(z,x,t\right)f\left(t,\theta\right)dt$, with $h\left(\cdot\right)$ as defined in lemma 6, and find conditions on f that guarantees that V satisfies SM with respect to θ - i.e., $d\left(V_{z}/V_{x}\right)/d\theta>0$. Once we find these conditions, we just have to notice that

$$\frac{d}{d\theta}\left(\frac{\partial V/\partial z}{\partial V/\partial x}\right)=\frac{1}{\partial V/\partial x}\left(\frac{\partial^{2}V}{\partial z\partial\theta}-\frac{\partial^{2}V}{\partial x\partial\theta}\frac{\partial V/\partial z}{\partial V/\partial x}\right)\geq 0 \tag{5}$$

At $\theta=\theta^{o}$, we know that $V_{z}=V_{x}$ which implies that the right hand side of (5) is just $d\left(V_{z}-V_{x}\right)/d\theta$.

Lemma 8 in Athey [1], states that $V\left(z,x,\theta\right)$ satisfies SM for all h that satisfy SM if and only if f is log-supermodular. Hence, the result. ■

Proof of Proposition 5. This is also a consequence of lemma 8 in Athey [1]. In fact, because f being log-supermodular and $h\left(\cdot\right)$ displaying SM are shown in her paper to be a minimal pair of sufficient conditions, then $V\left(z,x,\theta\right)$ will satisfy SM whenever f is log-supermodular if and only if $h\left(\cdot\right)$ satisfies SM. That is, if and only if z is normal. ■

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